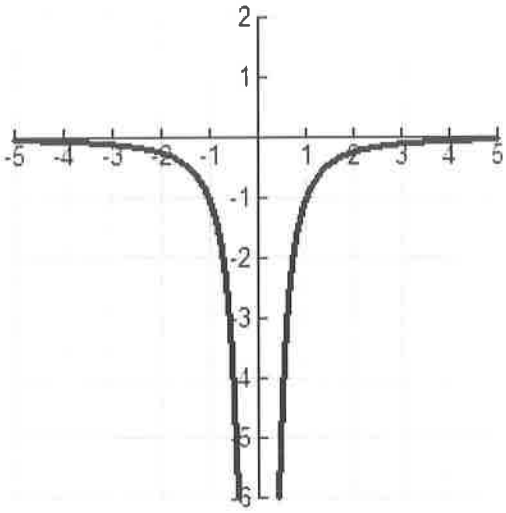
Math 4

Name Key

Semester 1 Exam Review

1. **Function Families**

Given a function or graph, find the domain, range, symmetries, degree and type of function family.



Domain: $X \neq 0$

Range: $Y < 0$

Symmetry: y-axis, $x=0$

Type of function: negative, inverse

2. **Function Operations**

Given two functions, calculate arithmetic operations and the composition of the functions.

$t(x) = 3x - 10$ $\quad$ $j(x) = x^2 + 2x + 5$

Calculate the following:

$[t+j](x)$

$(3x-10)+(x^2+2x+5)$

x^2+5x-5

$[t-j](x)$

$(3x-10)-(x^2+2x+5)$

$-x^2+x-15$

$t(x) \cdot j(x)$

$(3x-10)(x^2+2x+5)$

$3x^3-4x^2-5x-50$

$j(t(x))$

$(3x-10)^2+2(3x-10)+5$

$9x^2-54x+85$

3. **Parametric Equations**

Use parametric equation to construct a graph. Convert parametric equations to rectangular form.

$$\begin{cases} x(t) = t^2 - 3 \\ y(t) = 2t + 5 \end{cases}$$

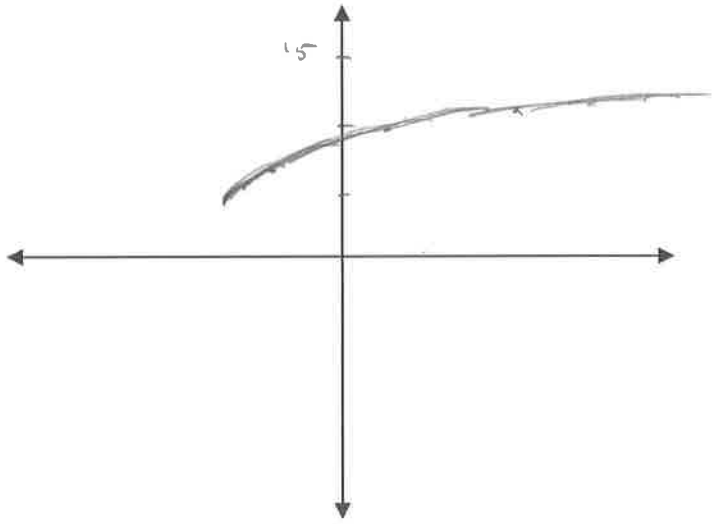
$x = t^2 - 3$

$t = \pm\sqrt{x+3}$

$y = 2(\sqrt{x+3}) + 5$

Graph the equations over the interval $0 \le t \le 5$ then convert the equations to rectangular form & simplify.

t	$x(t)$	$y(t)$
0	-3	5
1	-2	7
2	1	9
3	6	11
4	13	13
5	22	15



4. **Chunking/*u*-substitution**

Solve for x.

$$5e^{2x} - 9e^x - 10 = 2e^{2x} + 4e^x$$

$3e^{2x} - 13e^x - 10 = 0$

$u = e^x$

$3u^2 - 13u - 10 = 0$

$(3u + 2)(u - 5) = 0$

$u = -\frac{2}{3}$ $\quad$ $u = 5$

$e^x = -\frac{2}{3}$ $\quad$ $e^x = 5$

$x = \ln(-\frac{2}{3})$ $\quad$ $x = \ln(5)$

No solution

5. **Number Line Analysis**

Solve the inequality below.

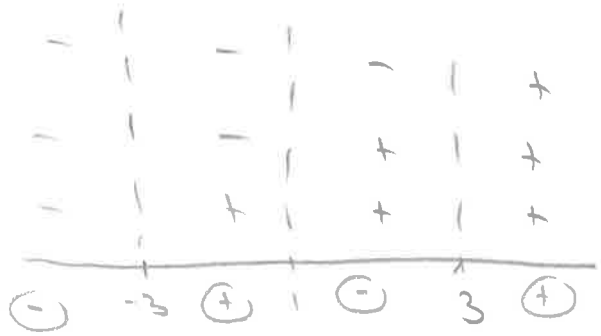
$$\frac{x^2 - 4x + 3}{x + 3} \le 0$$

$\frac{(x-3)(x-1)}{x+3} \le 0$

$x - 3$

$x - 1$

$x + 3$



$x < -3$ or $1 \le x \le 3$

6. Arithmetic and Geometric Sequences and Series including Sigma Notation

Write out what is meant by

$$\sum_{k=1}^{4} \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \frac{1}{9\cdot 11}$$

Express each of the following in sigma notation:

(a) $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

(b) $-1 + 2 - 3 + 4 - 5 + 6 - \ldots + 20$

a) $\sum_{n=1}^{5} \frac{1}{n}$ b) $\sum_{n=1}^{20} (-1)^n \cdot n$

Find the 12th term of the sequence 3, 8, 13, 18, ...

$S = 3 + 5(n-1)$

$S_{12} = 58$

Fill in the missing blanks of the arithmetic sequence 9, ___, ___, ___, 37

$d = \frac{37-9}{4} = 7$ 9, 16, 23, 30, 37

7. Matrix operations

Simplify each.

$$\begin{bmatrix} -1 & -1 \\ -6 & 3 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 6 \\ 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -17 & -37 \\ -16 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -6 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 3 \\ -2 \end{bmatrix}$$

Not possible

Use matrices to solve the system.

$$\begin{cases} -4x - 15y = -17 \\ -x + 5y = -13 \end{cases}$$

$$\begin{bmatrix} -4 & -15 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -17 \\ -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & -15 \\ -1 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -17 \\ -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

8. **Rational Functions**

1. Use limit notation to describe the behavior of $j(x)$ near its vertical and horizontal asymptotes. It would also be nice if you could draw a sketch of the function as well.

$$j(x)=\frac{2x+5}{x-3}$$

Horizontal

$\lim_{x\to\infty} j(x) = 2$

$\lim_{x\to-\infty} j(x) = 2$

Vertical

$\lim_{x\to 3^+} j(x) = \infty$

$\lim_{x\to 3^-} j(x) = -\infty$

2. Add, subtract, multiply or divide the rational expressions below. Simplify your answer.

a. $\dfrac{x^2-x-6}{2x^2+9x+4}\cdot\dfrac{x^2-16}{2x^2-7x-4}$

$\dfrac{(x-3)(x+2)}{(2x+1)(x+4)}\cdot\dfrac{(x+4)(x-4)}{(2x+1)(x-4)}$

$\dfrac{(x-3)(x+2)}{(2x+1)^2}$

b. $\dfrac{3x-4}{2x^2+3x+1}+\dfrac{5}{2x+1}$

$\dfrac{3x-4}{(2x+1)(x+1)}+\dfrac{5}{2x+1}\left(\dfrac{x+1}{x+1}\right)$

$\dfrac{8x+1}{(2x+1)(x+1)}$

c. $\dfrac{2(x+3)^2}{x-3}\div\dfrac{4}{x^2-9}$

$\dfrac{2(x+3)^2}{x-3}\cdot\dfrac{(x+3)(x-3)}{4}$

$\dfrac{(x+3)^3}{2}$

d. $\dfrac{6x+5}{2x+3}-\dfrac{2x-1}{2x+3}$

$\dfrac{4x+6}{2x+3}$

$\dfrac{2(2x+3)}{2x+3}$

2

Determine the values of the properties below. Write "none" if one does not exist. The domain is for the original function.

3. $f(x)=\dfrac{(2x-3)(x+5)}{x^2+6x+5}$

$=\dfrac{(2x-3)(x+5)}{(x+1)(x+5)}$

Domain: $x \neq -1, -5$

x-intercept(s): $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

horizontal asymptote(s): $y = 2$

vertical asymptote(s): $x = -1$

oblique asymptote: None

hole: $\left(-5, \frac{13}{4}\right)$

4. $w(x)=\dfrac{2x^2-13x-45}{3x^3+28x^2+9x}$

$=\dfrac{(2x+5)(x-9)}{x(3x+1)(x+9)}$

Domain: $x \neq 0, -\frac{1}{3}, -9$

x-intercept(s): $\left(-\frac{5}{2}, 0\right)$ $(9, 0)$

y-intercept: None

horizontal asymptote(s): $y = 0$

vertical asymptote(s): $x = 0, x = -\frac{1}{3}, x = -9$

oblique asymptote: None

hole: None

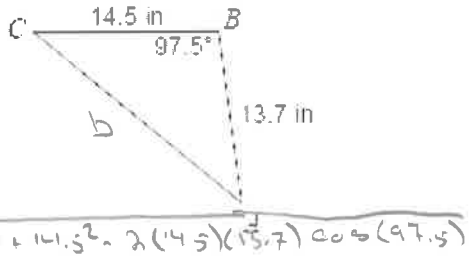
9. **Unit Circle**

Yeah, know all the values around the Unit Circle.

OK

10. **Law of Sines and Cosines**

Find $m\angle C$.



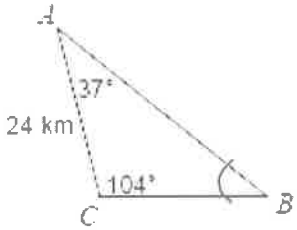
$b = \sqrt{13.7^2 + 14.5^2 - 2(14.5)(13.7)\cos(97.5)}$

$b = 21.2$

$\dfrac{21.2}{\sin 97.5} = \dfrac{13.7}{\sin C}$

$C = 39.8°$

Solve all missing sides and angles.



$\angle B = 39°$

$c = 37$ km

$a = 23$ km

$\dfrac{24}{\sin 39} = \dfrac{c}{\sin 104}$

$\dfrac{24}{\sin 39} = \dfrac{a}{\sin 37}$

11. **Trigonometric Simplification**

Remember: $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$, $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ and $\sin^2 x + \cos^2 x = 1$

*****As always, study your worksheets and tests/quizzes in addition to working through the problems in this review.**